Numerical simulations of water waves generated by subaerial granular and solid-block landslides: Validation, comparison, and predictive equations

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ABSTRACT

We compare waves generated by subaerial solid-block and granular landslides and propose equations for predicting their maximum initial wave amplitudes. The recent Anak Krakatau subaerial landslide tsunami in December 2018, which resulted in more than 450 deaths, demonstrated the knowledge gap on this subject and motivated this study. Here, we make numerical models using the numerical package FLOW3D-Hydro for solid-block and granular landslides and validate them using physical experiments. Results indicate that the maximum initial wave amplitudes generated by solid-block landslides are 107% larger than those generated by granular landslides in our experiments. The relationship between maximum initial wave amplitude and slope angle is inverse for solid-block slides whereas, it is direct for granular slides. However, a critical angle of 60° is achieved for granular slides, and for slope angles more than this critical value, the maximum wave amplitudes start to decrease. Regarding wave period, our results show that it remains nearly unchanged for both types of landslides as water depth and slide volume vary. The period generated by solid-block slides increases as the slope angle decreases; however, it remains unchanged for granular slides. The predictive equations are applied to real landslide tsunamis and resulted in satisfactory performances.

1. Introduction and literature review

The recent 2018 Anak Krakatau volcanic tsunami (Indonesia), which left a death toll of more than 450 people, once more highlighted the large tsunami hazards associated with subaerial volcanic collapses (Grilli et al., 2021; Satake et al., 2020; Omira and Ramalho, 2020; Muhari et al., 2019; Heidarzadeh et al., 2020). The large volume of rock materials, sliding into the water, generated destructive tsunami waves, which damaged most of the Sunda Strait coasts (Putra et al., 2020). Other destructive subaerial landslides that occurred in recent decades are the 1958 Lituya Bay event, where a landslide tsunami generated the largest-ever recorded tsunami with a runup of up to 524 m (Fritz et al., 2004), and the 1963 Vajont dam incident in Northern Italy, where an impulsive landslide-generated wave overtopped the dam and caused approximately 2000 casualties (Heller and Spinneken, 2013). The generation mechanism and characteristics of subaerial landslide tsunamis involve complex dynamic processes, involving interactions among three different phases (solid, liquid and air). These types of waves are generated by different geophysical phenomena such as landslides, slumps, debris flows, rockfalls, and snow avalanches and they can be either due to solid-block or granular failures (e.g., Ataie-Ashtiani and Yavari-Ramshe, 2011).

Several experimental studies have been conducted concerning subaerial landslide-generated waves including, 2D and 3D investigations (e.g., Fritz et al., 2004; Heller and Spinneken, 2015; Lavha et al., 2015; Jin et al., 2016; McFall and Fritz, 2016; Wang et al., 2017; Evers et al., 2019; Kim et al., 2020). In particular, the behaviour of landslide-generated impulsive waves, such as landslide motion, effective parameters, wave characteristics, and associated runup have been studied. Generally, analytical solutions have been available for some simple idealized cases (e.g., Noda, 1970; Pelinovsky and Poplavsky, 1996; Liu et al., 2003; Haugen et al., 2005). For more complicated cases, physical and numerical modelling have been applied. Numerical modelling plays an important role these days in studying landslide tsunamis due to their flexibilities and relatively low cost as compared to physical modelling (e.g., Fine et al., 2005; Lynett and Liu, 2005; Heller and Hager, 2011; Abadie et al., 2010; Heidarzadeh et al., 2019, 2022). Abadie et al. (2010) presented an idealized geometry for simulating...
the waves generated by subaerial landslides using a multi-fluid Navier-Stokes model. The slide viscosity was varied to investigate the effect of slide deformation on generated waves, which showed that the deformation of landslide shape could significantly impact the slide motion and waveforms. Ataie-Ashtiani and Yavari-Ramshe (2011) developed a 2D fourth-order Boussinesq-type model to study the impact of subaerial landslides in dam reservoirs considering the nonlinearity and frequency dispersion effects. This validated model was used to estimate the maximum wave amplitudes and runup of potential subaerial landslides for two real case studies. Ma et al. (2015) developed a model validated by comparing the results with analytical solutions for granular dam-break flows as well as experimental data. Lee and Huang (2021) studied the effect of grain size on subaerial landslide-generated waves through physical tests and numerical simulations. A multiphase flow model based on an Eulerian framework was employed to conduct the simulations. The results by Lee and Huang (2021), validated by physical measurements, revealed that finer sand materials slide faster and generate larger waves. Heller and Hager (2014) showed that various parameters may influence subaerial landslide-generated waves including slide width, slide length, slide thickness, slide volume, slide mass, water depth, slope angle, and landslide front angle. Our review of the literature reveals that few studies systematically compare landslide waves generated by solid-block and granular landslides; therefore, here we focus on such analysis.

In this research, we apply the FLOW3D-Hydro numerical package for modelling subaerial solid-block and granular landslides and validate them through physical experiments. This research was motivated in the aftermath of the 2018 Anak Krakatau tsunami where a wide range of initial tsunami wave amplitude (Fig. 1) was reported by different authors ($a_M = 100$ m by Paris et al., 2019; $a_M > 100$ m by Ren et al., 2020 and Heidarzadeh et al., 2020; $a_M = 40–100$ m by Zengaffinen et al., 2020). This implies that there are uncertainties in the prediction of the amplitudes of subaerial landslide tsunamis and the existing knowledge of this subject needs to be further developed. Therefore, this research is designed with two objectives: (i) To develop predictive equations for the maximum initial wave amplitudes generated by solid-block and granular subaerial landslides; and (ii) To perform a comparative study of the maximum amplitudes and waveforms generated by these two types of subaerial landslides. We generate new artificial data on the amplitude of subaerial landslide tsunamis through validated numerical models. In order to develop the new predictive equations, we make a large database of maximum wave amplitudes by combining our numerical data with existing experimental data in the literature.

2. Data and methods

Our dataset comprises existing experimental measurements combined with new artificial data generated through the validated numerical models of this study. We applied the numerical package FLOW3D-Hydro (version 1.0) which solves the fully three-dimensional transient Navier-Stokes equations using the fractional area/volume obstacle representation (FAVOR) and Volume of Fluid (VOF) methods (Flow Science, 2022; Kim et al., 2020; Sabeti and Heidarzadeh, 2021b). To compute flows at boundaries, FAVOR allows for the definition of solid boundaries within the Eulerian grid and determines fractions of areas and volumes open to flow in partially blocked volumes. Under the VOF method, the fluid volume fraction of each calculation unit is treated as a volume function that can effectively distinguish different phases. For efficiency, the TruVOF method is applied in FLOW3D-Hydro to capture the small amplitudes in the vertical direction (Sabeti and Heidarzadeh, 2021b). The governing equations for fluid flow are the Navier-Stokes equation given by:

$$\nabla \cdot \mathbf{u} = 0$$

(1)

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla P + \nu \nabla^2 \mathbf{u} + \mathbf{g}$$

(2)

where $\mathbf{u}$ is the velocity vector, $t$ represents time, $P$ is pressure, $\nu$ is the kinematic viscosity, and $g$ is the gravitational acceleration.

2.1. Physical modelling of subaerial landslide-generated waves

In order to validate our numerical models, two sets of physical experiments for solid-block and granular landslides were conducted in a rectangular water flow flume 4.0-m in length, 0.26-m in width and 0.5-m in height at the Brunel University London (UK). For both solid-block and granular landslides in our physical experiments, the slope angle ($\alpha$) and water depth ($h$) were fixed at 45$^\circ$ and 0.246 m, respectively (Figs. 1 and 2a). The sliding process was recorded by a digital camera (i.e. Sony A6300) with a sampling frequency of 120 frames per second, which measured the travel time ($t_t$). As the travel distance was kept constant for both solid-block and granular landslides ($D = 0.045$ m), the acceleration of the slides ($a_s$) can be obtained by the equation, $a_s = 2D/t_t^2$ (Xue et al., 2019). We note that, by ‘travel distance’, we mean the

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Fig. 1. Sketch showing the parameters used in this study for modelling subaerial solid-block and granular landslides. Parameters are: $h$, water depth; $a_M$, maximum wave amplitude; $\alpha$, slope angle; $l_s$, length of landslide; $D$, travel distance (the distance from toe of the sliding mass to the water surface); $\Delta z$, the drop height measured between the locations of the landslide centroid at rest and the landslide centroid reaching the initial water level; SWL, still water level; and $H$, trough-to-crest wave height.
distance from toe of the sliding mass to the water surface (Fig. 1) throughout this research. The landslide impact velocity ($v_s$) could be calculated by the equation,

$$v_s^2 = 2a_s D.$$  

The measured landslide velocities in our physical experiments were approximately consistent with the values calculated using energy balance between the slide release and the impact location (Fritz et al., 2004):

$$v_s = \sqrt{2g\Delta z(1 - \tan \delta \cot \alpha)}$$  

(3)

where $g$ is the gravitational acceleration, $\Delta z$ is the drop height (Fig. 1) ($\Delta z = 0.23$ m for granular material and $\Delta z = 0.28$ m for solid block in our physical experiments), $\alpha = 45^\circ$ is slope angle, and $\delta$ is dynamic bed friction angle between slide bottom and slope surface ($\delta = 24^\circ$ for solid blocks, and $\delta = 28^\circ$ for granular material).

Table 1 gives detailed information about the dimensions and kinematics of the solid-block and granular slides which were employed in our physical experiments. For solid-block landslides, we tested only one particular shape (a triangular prism). Natural Granite materials were used for granular landslides with the grain density of 2750 kg/m$^3$, an average grain diameter of 10 mm, bulk slide density of 1680 kg/m$^3$ and bulk slide porosity of 15%. A concrete prism block with a density of 2600 kg/m$^3$ was used to produce landslide-generated waves by solid blocks. The volumes of the solid-block and granular slides were the same ($V = 2.60 \times 10^{-3}$ m$^3$). The net weights of the solid-block and the granular material landslides were 6.86 kg and 6.02 kg, respectively, indicating a 13% weight difference between them. Ataie-Ashtiani and Nik-Khah (2008) showed that the shape of the sliding mass has negligible impacts on the dynamic parameters of the landslide-generated waves. Tang et al. (2018), by changing mean grain size from 10 mm to 30 mm, reported up to 15% of changes in maximum landslide-generated wave amplitudes. Heller and Hager (2014) were able to estimate wave amplitudes of subaerial landslides with high...
Geometrical information of the slides used for numerical simulations in this study.

<table>
<thead>
<tr>
<th>Parameter, unit</th>
<th>Solid block</th>
<th>Granular material</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slide width ($b_i$), m</td>
<td>0.260</td>
<td>0.260</td>
</tr>
<tr>
<td>Slide length ($l_i$), m</td>
<td>0.200</td>
<td>0.200</td>
</tr>
<tr>
<td>Slide thickness ($s_i$), m</td>
<td>0.100</td>
<td>0.100</td>
</tr>
<tr>
<td>Slide volume ($V_i$), m$^3$</td>
<td>2.60 × 10$^{-3}$</td>
<td>2.60 × 10$^{-3}$</td>
</tr>
<tr>
<td>Specific gravity ($\gamma_i$)</td>
<td>2.600</td>
<td>2.750</td>
</tr>
<tr>
<td>Slide weight ($m_i$), kg</td>
<td>6.860</td>
<td>6.020</td>
</tr>
<tr>
<td>Slide velocity ($v_i$), m/s</td>
<td>1.816</td>
<td>1.310</td>
</tr>
<tr>
<td>Slide Froude number$^b$ ($F_i$)</td>
<td>1.165</td>
<td>0.836</td>
</tr>
</tbody>
</table>

$^a$ The weights of the solid-block and granular material are slightly different because of the larger porosity of the granular material.

$^b$ Slide Froude number is calculated by using the following equation: $F_i = v_i / \sqrt{gh}$; the water depth ($h$) is 0.246 m in our physical experiments.

The accuracy without considering the effect of grain size. It was also reported by Tang et al. (2018) that the landslide front angle might slightly impact the wave amplitude for slope angles ($\alpha$) larger than 30°. Because our experiments are considered those with large hill slope angles (30° - 65°), the impact of landslide front angle is assumed negligible.

The scale effect was considered in our physical experiments. The criterion for avoiding scale effects was proposed by Heller et al. (2008), who reported that the scale effects can be negligible for cases where the Reynolds number ($R = \rho V h^2 / \mu$; where $\mu$ is kinematic viscosity) is more than 3.0 × 10$^5$ and the Weber number ($W = \rho V h^3 / \sigma$; where $\sigma$ is surface tension coefficient) is more than 5.0 × 10$^5$ or water depth ($h$) is approximately above 0.20 m. We considered the kinematic viscosity of water as $\nu = 1.01 \times 10^{-6}$ m$^2$/s and surface tension coefficient of $\sigma = 0.073$ N/m at 20°C (Xue et al., 2019). Given these parameters and data of our physical experiments (Table 1), we obtained $R = 3.8 \times 10^5$ and $W = 8.1 \times 10^5$, which indicates that the scale effect can be insignificant in our experiments. Wave measurements during our laboratory experiments were conducted using a twin wire wave probe from HR Wallingford products (HRIA-1016: https://equipt.hrwallingford.com/hydraulic-lab-equipment/wave-gauge-system).

### 2.2. Numerical simulations of subaerial landslide-generated waves

For numerical modelling, the entire flow domain was 0.26 m wide, 0.50 m deep and 4.0 m long (Fig. 2). The fluid inside the flume was specified with a water depth of 1000 kg/m$^3$ at 20°C. The water depth ($h$) was varied within 0.150–0.246 m. The slope angle ($\alpha$) was varied from 30° to 60°. Six solid blocks and six granular materials with the shape of a prism, having variable volumes (0.70 × 10$^{-3}$ m$^3$ – 2.60 × 10$^{-3}$ m$^3$), were used in our simulations to provide a range for landslide volumes (Table 2). The landslide width is fixed at 0.26 m for all landslides (Table 2). During the validation stage of the numerical model, the water depth was 0.246 m, and the slope angle was 45° for both types of landslides (Fig. 2c and d). The specific gravity ($\gamma_i$) for solid blocks was 2.60 and for granular material was 2.75, consistent with actual landslides in our physical experiments. For each type of landslide, a uniform grid comprising of one single mesh plane was used to solve the equations. For solid blocks, a grid size of 0.0020 m was applied in an area of 4.0 m (x-direction) × 0.5 m (z-direction), whereas a grid size of 0.0010 m was used for granular material in the same area. The total number of computational cells for solid-block landslides was 3.75 × 10$^6$; this value for granular landslides setup was 15 × 10$^5$. It is noted that although the model is in 2D, it is advised to consider at least a few cells in the width direction ($y$-direction) (Flow Science, 2022); we considered 10 cells in $y$-direction. The computational time for a granular landslide simulation is approximately 3.5 h on a PC Intel (R) Core (TM) i7-8700 CPU with a frequency of 3.20 GHz equipped with a 32 GB Ram.

The top, front and back of the mesh areas were defined as symmetry, and the other surfaces were of wall type with no-slip conditions around the walls. The symmetry boundary condition indicates that the conditions outside of the boundary are identical to those inside the boundary. The Renormalization Group (RNG) model was used for simulating turbulent flows because of its wider applicability and for its accuracy in modelling turbulent flows (Choi et al., 2007; Sabeti and Heidarzadeh, 2021b). In the simulations presented herein, the landslide movement has been reproduced by the coupled motion object, which means instead of prescribed motion where the force and torques should be given to the model, the movement of landslides is based on the gravity and friction between surfaces. The time intervals of the outputs of the numerical model were set to 0.02 s in order to be consistent with the actual sampling rates of our wave gauges in the laboratory. As the largest value for maximum initial wave amplitude occurred approximately at the distance of 0.40 m from the toe of the slope, this location was chosen for the installation of the wave gauge (Fig. 2a and b).

The friction coefficient ($f$) in the coupled motion is designated as 0.45 based on Coulombic friction measurements in the laboratory to calibrate the model for solid-block landslides, and it was 0.53 for granular landslides. We note that friction coefficient ($f$) is related to the dynamic bed friction angle ($\delta$) through the following equation:

$$f = \tan(\delta) \quad (4)$$

As the Coulombic friction coefficient changes relatively in a small domain, most existing studies have not incorporated it as an independent variable in the predictive equations (Fritz et al., 2004; Heller and Spinnken, 2015; Noda, 1970). The current collective understanding from literature is that this practice is not likely introducing significant errors.

The Courant Number ($C$) is used as a metric for FLOW3D-Hydro solver to ensure the stability of the numerical calculations and to evaluate the suitability of the time step ($\Delta t$) during simulations for a particular mesh size ($\Delta x$) and flow speed ($U$). The Courant number, which is always kept sufficiently below one, is given by the following equation:

$$C = \frac{U \Delta t}{\Delta x} \quad (5)$$

where $C$ is Courant Number, and $\Delta t$ is the time step. FLOW3D-Hydro employs a dynamic time step ($\Delta t$) for simulations, which means the time step sizes are dynamically adjusted during a transient simulation according to the convergence behaviour of the nonlinear iteration scheme. In our simulations, the initial time step was 0.0012 s and varied in the range of 0.00015–0.00075 s during the simulations.

Validation of the numerical models is performed through the comparison between numerical results and laboratory experimental data (Fig. 2). In order to evaluate the quality of the agreement between laboratory observations and numerical simulations, we used the following criterion:

Table 1
Geometrical and kinematic information of the solid-block and granular landslides used for physical experiments in this study.

<table>
<thead>
<tr>
<th>Parameter, unit</th>
<th>Solid block</th>
<th>Granular material</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slide Froude number$^b$ ($F_i$)</td>
<td>1.165</td>
<td>0.836</td>
</tr>
</tbody>
</table>
5

\[ \varepsilon = \frac{\text{Obs} - \text{Sim}}{\text{Obs}} \times 100 \]  

(6)

where \( \varepsilon \) is the mismatch error, \( \text{Obs} \) is the laboratory observation values, and \( \text{Sim} \) is the simulation values. Our results revealed that the mismatch errors between physical experiments and numerical models for the solid-block and granular material are 5% and 6%, respectively, indicating that our models reproduce the measured waveforms satisfactorily (Fig. 2 e-f). For spectral analysis of the water waves, we applied the Fast Fourier Transform (FFT) of MATLAB package (MathWorks, 2022).

In order to find the most efficient mesh resolution, mesh sensitivity analyses were conducted for the solid-block and granular landslides (Fig. 3) (e.g., Heidarzadeh and Feiz, 2022; Heidarzadeh et al., 2017). The initial size of the mesh for the granular slides was twice smaller than that of the solid-block slides to ensure that the model captures the smaller wave amplitudes generated by granular materials. In both types of landslides, we considered the influence of mesh density on simulated waveforms by doubling and halving the current meshes. The results revealed that the coarser mesh is deviating 4% (Fig. 3a, \( \Delta x = 0.0040 \text{ m} \)) from the two other finer meshes for the solid block; this deviation was 5% for the granular landslide (Fig. 3a, \( \Delta x = 0.0020 \text{ m} \)). As the simulation results from the two finer meshes for both solid-block and granular materials do not show any improvements, the current mesh sizes (\( \Delta x = 0.0020 \text{ m} \) for solid block, and \( \Delta x = 0.0010 \text{ m} \) for granular material) are suitable and are therefore used for our simulations.

2.3. Analysis of landslide velocity

The mechanism of landslide motion both under air and underwater is discussed by Panizzo et al. (2005) detailing that the motion starts from the rest position and accelerates under gravitational forces until it impacts the water, which causes a rapid decrease in slide velocity until it stops. Fig. 4 shows the displacement and velocity profiles of the landslides during the entire travel of the landslides. These profiles are based on landslide block number 6 (Slide-6 in Table 2), at the water depth of \( h = 0.246 \text{ m} \), and slope angle of 45°. The velocity of the slide (\( v_i \)) reaches its maximum value at the time of intrusion into water, which is called the impact time (\( t_{\text{imp}} \)). After the impact, the submarine motion of the slides occurs until they stop (\( t_{\text{stop}} \)). In order to measure the displacement and velocity of the slides, we attached a probe to the centre of each landslide (one for solid-block, and one for granular slide) in our numerical models, which measured the displacement and velocity of the landslides from the release point until they stopped. Fig. 4 shows that the solid-block landslide reaches a maximum velocity of 1.8 m/s whereas the granular slide achieves a maximum velocity of 1.35 m/s. It takes approximately 1.0 s for the solid landslide to come to rest while the granular landslide is at motion for approximately 1.3 s. It is noted that the landslide displacement in Fig. 4 is expected to constantly increase by time. Such a trend is generally observed for both types of landslides (Fig. 4) although the displacement curve of the granular landslide is associated with some oscillations, which is attributed to the chaotic movements and interactions of the grains during the granular landslide (Fig. 4b).

2.4. Data for developing the predictive equation

To develop our predictive equations, we benefit from published experimental data on both solid-block and granular landslides (Fig. 5), in addition to new numerical data generated by our validated models. The published data belong to Zweifel (2004), Saelevik et al. (2009), Miller et al. (2017), and Xue et al. (2019). All these experiments are based on 2D laboratory tests in wave flumes with lengths of 11 m, 25 m, 33 m and 10 m, respectively. Zweifel (2004) studied subaerial landslide dynamics through physical modelling and reported that water depth and slide thickness are the governing parameters on the maximum wave amplitudes. Saelevik et al. (2009) focused on the effect of slide geometry on impulsive waves generated by subaerial landslides. The results reported by Saelevik et al. (2009) indicated that, by decreasing the slide length, a deeper void for the collapse of leading wave is created, and therefore a larger trailing wave is generated. Miller et al. (2017) conducted a series of large-scale physical modelling (a flume with length of 33 m) and studied the impact of effective mass (a fraction of total landslide mass) for activating the leading waves. Fig. 5 gives a summary of the previously-published data along with our data whereas Fig. 6 presents the wave classifications for data generated in this study as well as some of the data from literature for which information was available. Fig. 6 indicates that all data points belong to intermediate waves and majority of them are of Stocks type.

3. Results

We apply our validated numerical models to simulate the subaerial landslide-generated waves by two types of slides, i.e. solid-block and granular. The sensitivities of wave amplitudes are investigated relative to four main landslide parameters comprising, water depth (\( h \)), landslide volume (\( V \)), slope angle (\( \alpha \)), and landslide thickness (\( f \)). Previous studies revealed that the aforementioned parameters have the largest impacts on the wave amplitudes generated by landslide tsunamis (Kamphuis and Bowering, 1976; Heller and Hager, 2014; Sabeti and Heidarzadeh, 2020, 2021a). The travel distance (\( D \)) was kept constant for all these
3.1. The effects of water depth ($h$) on wave amplitude

In order to investigate the impact of water depth ($h$) on maximum initial wave amplitude ($a_M$), water depth was varied between 0.150 m and 0.246 m for solid-block and granular landslides (Fig. 7). The slope angle, slide volume and travel distance were kept constant for this series of tests (i.e., $\alpha = 45^\circ$, $D = 0.02$ m and $V = 2.60 \times 10^3$ m$^3$) (Fig. 7). To minimize the scale effects on our models, we ensured that water depth is above 0.150 m which is approximately close to the criteria proposed by Heller et al. (2008). The results showed that for both types of landslides, the maximum amplitude increases by a decrease in water depth as previously reported by other authors (Zweifel et al., 2006; Oppikofer et al., 2019). The highest value of $a_M$ for solid-block landslide occurred at the lowest water depth ($a_M = 0.086$ m at $h = 0.15$ m). The other crucial finding was that the maximum amplitude generated by solid blocks are approximately twice larger than that produced by granular materials, e.g., at the same water depth of 0.240 m, $a_M = 0.07$ m for a solid-block slide while it is $a_M = 0.036$ m for the granular slide (Fig. 7b). By comparing all maximum amplitudes from solid-block and granular slides, it was found that maximum amplitudes generated by solid-block slides are approximately 92% larger than those produced by granular landslides, on the average. Spectral analyses of the waveforms revealed that the wave period remains approximately the same for both solid-block and granular landslides in our experiments (Fig. 7c).

3.2. The effects of landslide volume ($V$) on wave amplitude

A series of simulations were conducted to study the influence of slide volume ($V$) on wave amplitudes and waveforms. Fig. 8 gives the simulated waveforms for varying volumes in the range of $0.70 \times 10^3$ m$^3$–$2.60 \times 10^3$ m$^3$. The slope angle, water depth and travel distance were kept constant for this series of analyses (i.e., $\alpha = 45^\circ$, $h = 0.246$ m and $D = 0.02$ m) (Fig. 8). The results demonstrate that by increasing slide volume, the maximum initial wave amplitude increases for both solid blocks and granular landslides, which is consistent with published results in the literature (e.g., Kamphuis and Bowering, 1970; Wang et al., 2017). The highest maximum initial wave amplitude ($a_M = 0.045$ m) was recorded for the largest solid block ($V = 2.60 \times 10^3$ m$^3$). In addition, we observed that the maximum amplitudes generated by solid blocks are approximately twice larger than those from granular materials for the same slide volume. On the average, the maximum amplitudes from solid-block slides are 121% larger than those from granular slides. Similar to the effect of water depth studied in the previous section, here we witness that the wave period remains approximately unchanged for both solid-block and granular landslides, as evidenced by our spectral analyses (Fig. 8c).

3.3. The effects of slope angle ($\alpha$) on wave amplitude

The effect of slope angle on wave amplitude was analyzed by simulating models with different slope angles ($30^\circ$–$60^\circ$) for both types of landslides (Fig. 9). The water depth, travel distance and slide volume were kept constant for this series of simulations (i.e., $h = 0.246$ m, $D = 0.02$ m and $V = 2.60 \times 10^3$ m$^3$). For solid-block landslides, the numerical simulations revealed that there is a linear relationship between simulations. In the following, the results are discussed in separate subsections.
Fig. 5. Experimental data from previously-published researches on subaerial solid-block and granular landslides along with our artificial numerical data, which are used for developing our predictive equations. Abbreviations are XUE-2019 for Xue et al. (2019); ZWL-2004 for Zweifel (2004); SLK-2009 for Sælevik et al. (2009); and MLR-2017 for Miller et al. (2017). Parameters are: 2019 for Xue et al. (2019); ZWL-2004 for Zweifel (2004); SLK-2009 for Sælevik et al. (2009); and MLR-2017 for Miller et al. (2017). Parameters are: h, water depth; V, slide volume; α, slope angle; and α0, the maximum initial wave amplitude. For Miller et al. (2017), we used the non-breaking wave data.

3.4. The effects of dynamic bed friction coefficient (f) on wave amplitude

In order to study the effect of friction coefficient (f) on maximum initial wave amplitude (α0), we changed f in the range of 0.30–0.55 (Fig. 11). This range is consistent with previous reports on the values of dynamic bed friction angle (δ), which is reported as being 20°–30° (Mohammed and Fritz, 2012; Lindstrom, 2016; McFall and Fritz, 2016); such values for δ would result in f = 0.36–0.58 according to Equation (4). The results indicate that α0 is not strongly affected by friction coefficient for both types of landslides (i.e., solid-block, and granular slide). According to Fig. 11, the maximum wave amplitudes are affected less than 14% due to changes of the friction coefficient in the range of f = 0.30–0.55.

3.5. Wave velocity analysis

Fig. 12 shows a sequence of three water particle velocity plots at different times during landslide motion. The direction and magnitude of the velocity are shown by the direction and the length of the arrows (Fig. 12). A combination of inward and upward flows forms a complicated wavefield around the landslide generation area, which explains the splashing of water and mixing with the air around the source zone (Fig. 12). The sequence of solid-block landslides starts at t = 0.180 s after landslide impact and covers two other snapshots at 0.34 s and 0.48 s from the onset of the landslide (Fig. 12a–c). For the granular landslide, the landslide front profile, in the numerical image of Fig. 12d, is almost vertical as the materials start to enter into water, followed by a switch from rigid to fluid motion. The results in Fig. 12 indicate that, for the case of a landslide in a water depth of 0.246 m, a slope angle of 45°, a slide volume of 2.60 × 10^{-3} m^3, the highest velocity magnitude for the solid-block landslides is 1.62 m/s whereas it is 0.92 m/s for the granular slide. This difference in wave velocity magnitude could be due to the deformation of granular slides whereas solid-block landslides do not deform. For the slide volumes V1 (0.70 × 10^{-3} m^3) to V6 (2.60 × 10^{-3} m^3) (Table 2), we observe that the maximum water particle velocity ranged from 0.98 m/s to 1.62 m/s for solid-block landslides. This velocity varied in the range of 0.68 m/s - 0.92 m/s for granular slides.
Predictive equations for landslide-generated waves can be helpful towards preliminary hazard assessment (Heller and Spinneken, 2015; Murty, 2003). Empirical equations which require a few landslide parameters can deliver rapid and straightforward estimates of the potential hazards associated with landslides, in particular as pre-existing information on landslide geology and rheology are usually limited. Here, we use a combination of existing experimental data from literature together with new numerical data generated by our 50 simulations to develop two new predictive equations for solid-block and granular landslides. We consider four main landslide parameters which are slope angle, slide volume, water depth and landslide thickness, by taking into account the fact that available information on landslide parameters is

Fig. 7. a) Waveforms of landslide-generated waves for solid-block (solid lines) and granular (broken lines) landslides for varying water depths in the range of \( h = 0.222 - 0.246 \) m. b) Same as ‘a’ but for water depths of \( h = 0.150 - 0.216 \) m. c) Spectra for a few of the waveforms. “Solid” and “Gran” in the legends represent solid-block and granular landslides, respectively. \( \eta \) is wave amplitude.
usually limited. Although incorporation of a few landslide parameters in the equations and absence of landslide velocity may introduce some uncertainties in the equations, it is believed that such uncertainties would be negligible because, in reality, most landslides move under gravity and thus velocity is a predictable parameter.

The curve-fitting on our simulation data indicates that for both types of landslides, \( V \) is directly correlated with \( a_M \) (Fig. 8) whereas \( h \) is inversely related to \( a_M \) (Fig. 7). The relationship between slope angle and \( a_M \) for solid-block landslides are inverse, however, this correlation is direct for granular material up to the critical slope angle of \( \alpha = 40^\circ \) – \( 60^\circ \), based on previous studies and our own results. By considering the non-dimensional forms of these four essential landslide parameters, here, two equations are developed for predicting \( a_M \) as follows.

For solid-block landslides, \( a_M \) is given by the following equation:

\[
\text{For solid-block landslides, } a_M \text{ is given by the following equation:}
\]

\[
\text{Fig. 8. a) Waveforms of landslide-generated waves for solid-block (solid lines) and granular (broken lines) landslides for varying slide volumes in the range of } V = 1.30 \times 10^{-3} \text{ m}^3 – 2.60 \times 10^{-12} \text{ m}^3. \text{ b) Same as ‘a’ but for slide volumes } V = 0.70 \times 10^{-3} \text{ m}^3 – 1.00 \times 10^{-3} \text{ m}^3. \text{ c) Spectra for a few of the waveforms. “Solid” and “Gran” in the legends represent solid-block and granular landslides, respectively. } \eta \text{ is wave amplitude.}
\]
For granular landslides, we derived the following equation:

$$a_M / h = 0.2152 \ (\tan \alpha)^{0.244} \left( \frac{V}{h^3} \right)^{0.601} \left( \frac{h}{s} \right)^{-0.174}$$  \hspace{1cm} (8)$$

where $a_M$ is maximum initial wave amplitude in meters, $\alpha$ is slope angle in degrees, $V$ is slide volume in m$^3$, $s$ is slide thickness in meters, and $h$ is water depth in meters. We note that our study on the effects of friction coefficient ($f = 0.3$–$0.55$) on the wave amplitudes (Fig. 11) revealed that the amplitudes would change up to 10% and 14% for solid-blocks and granular materials, respectively. Therefore, we assume uncertainty ranges of $\pm 10\%$ and $\pm 14\%$ for Equations (7) and (8), respectively.

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**Fig. 9.** a) Waveforms of landslide-generated waves for solid-block (solid lines) and granular (broken lines) landslides for varying slope angles $\alpha = 55^\circ - 65^\circ$. b) Same as ‘a’ but for slope angles $\alpha = 30^\circ - 50^\circ$. c) Spectra for a few of the waveforms. “Solid” and “Gran” in the legends represent solid-block and granular landslides, respectively. $\eta$ is wave amplitude.
which are in an acceptable range as per engineering experiments. Fig. 13 presents the performance of these two equations towards the prediction of experimental data which indicates satisfactory performances as the data points are aligned around the 45° lines. In order to further examine the performance of our predictive equations, we applied them to actual real-world subaerial landslide tsunami events: the 2018 Anak Krakatau (solid-block slide), the 1792 Unzen (granular slide), and the 1958 Lituya Bay (granular slide) incidents (Table 3). According to Table 3, our predictive equations successfully reproduce the initial maximum amplitudes of the Lituya Bay and the Anak Krakatau events, but the results show some uncertainties for the 1792 Unzen event. Regarding the application of the Equations (7) and (8), granular landslides are those involving granular formations including cobble, gravel, sand and soil with grain sizes in the range of 1 mm–100 mm. Solid-block landslides refer to those with large rock, stone and boulder units (https://www.usgs.gov/programs/landslide-hazards/landslides-101).

4. Comparison with previously-published predictive equations

Here, we compare the performance of our equations (Eqs. 7 and 8) with other previously-published relationships for the prediction of maximum initial wave amplitudes of subaerial solid-block and granular landslides (Table 4). For the granular slide of the 1958 Lituya Bay event with a reported amplitude of 152 m, the equations in Table 4 yield predictions in the range of 129–328 m (error ranges of 0–116%). The three best results are achieved using the equations of Fritz et al. (2004), that of this study (Eq. 8), and that of Robbe-Saule et al. (2021). The predictions for the solid-block event of 2018 Anak Krakatau (observed amplitude of 134 m) lie in the range of 31–5762 m implying an error domain of 0–4200% (Table 4). The two equations with the best results are that of this study (Eq. 7) and the relationship by Noda (1970). For the event of 1792 Unzen with a reported wave amplitude of 10 m, our equation predicts an amplitude of 22 m whereas the prediction made by other equations is in the range of 4.2–175 m (Table 4).

Compared to other predictive equations available in the literature, our two predictive relationships (Eqs. 7 and 8) perform well in reproducing actual subaerial landslide events although both of them use only four landslide parameters (i.e., volume, water depth, slope angle, and landslide thickness). It is noted that the equation by Fritz et al. (2004) has fewer number of parameters than our equations and gives a better prediction for the 1958 Lituya Bay event, however it underperforms for the case of the 1792 Unzen event (Table 4). The fewer parameters required for the application of our predictive equations is an advantage and would facilitate preliminary tsunami hazard assessment for vulnerable locations. It is worth noting that the success of our predictive equations is the result of continuous efforts in the past two decades in predicting landslide tsunamis as we benefited from a large amount of existing laboratory data in the process of deriving our equations (Fig. 5).
5. Conclusions

We applied the numerical package FLOW3D-Hydro to model subaerial solid-block and granular landslides and validated them through physical experiments in order to study the characteristics of waves generated by these two types of slides and to develop predictive equations. The volumes of both types of landslides were kept the same and they were approximately of similar weights as well (a weight difference of 13%). The main findings are:

- The maximum initial wave amplitudes for subaerial solid-block slides were 107% larger than those for granular slides, for the same slide volumes and approximately similar weights.
- An inverse relationship was observed between slope angle and the maximum initial wave amplitudes of subaerial solid-block landslides where the maximum amplitudes linearly decrease as the slope angle increases. However, for granular slides, a direct relationship was detected up to a critical slope angle of 60° from where maximum amplitudes start to decrease for slope angles more than 60°.
- Experiments revealed that wave periods remain nearly unchanged as water depth and slide volume vary for both subaerial solid-block and granular landslides. However, it was observed that the wave period increases as the slope angle decreases for solid-block slides. Such a change was not seen for granular landslides.
- The maximum water particle velocity for subaerial solid-block landslides was significantly larger than that for granular slides; for example, for the case of a landslide in a water depth of 0.246 m, a slope angle of 45° and a slide volume of $2.60 \times 10^{-3} \text{ m}^3$, the maximum velocity was 1.62 m/s for the solid-block whereas it was 0.92 m/s for the granular landslide.
- We proposed two new predictive equations for the maximum initial wave amplitudes of subaerial solid-block and granular landslides,

![Fig. 12. Snapshots of simulations at different times for solid-block (a, b and c) and granular landslides (d, e and f) showing magnitudes and directions of particle velocity (colour maps and arrows). The colormaps indicate water particle velocity in m/s and the arrows show the directions and magnitudes of water particle movements. (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)](image1)

![Fig. 13. Performance of the developed predicative equations ($a_{M_{\text{cal}}}$, Eqs. (7) and (8)) in reproducing experimental data ($a_{M_{\text{obs}}}$).](image2)
Comparing the performance of our newly-developed equations with the existing equations for predicting the amplitudes of actual worldwide subaerial landslide events. Parameters are: slide thickness ($s$), water depth ($h$), landslide volume ($V$), and maximum initial wave amplitude ($a_{0a}$).

Table 3
<table>
<thead>
<tr>
<th>Event</th>
<th>Landslide type</th>
<th>$V$ (m$^3$)</th>
<th>$h$ (m)</th>
<th>$s$ (m)</th>
<th>Slope angle ($\phi$)</th>
<th>Observed $a_{0a}$ (m)</th>
<th>Calculated $a_{0a}$ (m)</th>
<th>Equation number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lituya Bay (1958)$^a$</td>
<td>Granular</td>
<td>$3.06 \times 10^7$</td>
<td>122</td>
<td>120</td>
<td>45</td>
<td>152$^a$</td>
<td>144</td>
<td>8</td>
</tr>
<tr>
<td>Unzen (1792)$^b$</td>
<td>Granular</td>
<td>$34.0 \times 10^7$</td>
<td>2900</td>
<td>400</td>
<td>10</td>
<td>$10^b$</td>
<td>22</td>
<td>8</td>
</tr>
<tr>
<td>Anak Krakatau (2018)$^c$</td>
<td>Solid-block</td>
<td>$2.11 \times 10^7$</td>
<td>50</td>
<td>114</td>
<td>45</td>
<td>134$^c$</td>
<td>126</td>
<td>7</td>
</tr>
</tbody>
</table>

$^a$ Heidarzadeh et al. (2020), Grilli et al. (2019) and Grilli et al. (2021).
$^b$ Yavari-Ramshe and Ataie-Ashtiani (2016), Wang et al. (2019), and Miyamoto (2010).
$^c$ Fritz et al. (2004).

Table 4
Comparing the performance of our newly-developed equations with the existing equations for predicting the amplitudes of actual worldwide subaerial landslide events. Parameters are $a_{0a}$, initial maximum wave amplitude; $h$, water depth; $\rho_s$, slide density; $\rho_w$, water density; $v_s$, slide velocity; $V$, slide volume; $b_s$, slide width; $l_s$, slide length; $s$, slide thickness; $\alpha$, slope angle; $m_l$, slide mass; and, $V_{wm}$, the volume of the final immersed landslide. We considered $V_{wm} = V$ for both types of landslides.

<table>
<thead>
<tr>
<th>Type</th>
<th>Predictive equations$^d$</th>
<th>Author (year)</th>
<th>Observed $a_{0a}$ (m)</th>
<th>Calculated $a_{0a}$ (m)</th>
<th>Error, $\varepsilon$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Granular landslide (1958 Lituya Bay)</td>
<td>$a_{0a}/h = 10^{-1.25 \pm 0.71} \left( \frac{V}{h^2} \right)^{0.5} \left( \frac{\rho_s}{\rho_w} \right)^{0.5} \left( \frac{v_s}{s} \right)^{0.5}$</td>
<td>Slingerland and Voight (1982)</td>
<td>152</td>
<td>328</td>
<td>116</td>
</tr>
<tr>
<td>Granular landslide (1792 Unzen)</td>
<td>$a_{0a}/h = 0.25 \left( \frac{V}{h^2} \right)^{0.5} \left( \frac{\rho_s}{\rho_w} \right)^{0.5} \left( \frac{v_s}{s} \right)^{0.5}$</td>
<td>Fritz et al. (2004)</td>
<td>152</td>
<td>152</td>
<td>0</td>
</tr>
<tr>
<td>Granular landslide (1958 Lituya Bay)</td>
<td>$a_{0a}/h = 4 \left( \frac{V}{h^2} \right)^{0.5} \left( \frac{\rho_s}{\rho_w} \right)^{0.5} \left( \frac{v_s}{s} \right)^{0.5}$</td>
<td>Heller and Hager (2014)</td>
<td>152</td>
<td>216</td>
<td>42</td>
</tr>
<tr>
<td>Granular landslide (1792 Unzen)</td>
<td>$a_{0a}/h = 0.25 \left( \frac{V}{h^2} \right)^{0.5} \left( \frac{\rho_s}{\rho_w} \right)^{0.5} \left( \frac{v_s}{s} \right)^{0.5}$</td>
<td>Robbe-Saule et al. (2021)</td>
<td>152</td>
<td>129</td>
<td>15</td>
</tr>
<tr>
<td>Solid-block landslide (2018 Anak Krakatau)</td>
<td>$a_{0a}/h = 2.152 \left( \frac{\tan \theta}{\tan \phi} \right)^{0.444} \left( \frac{V}{h^2} \right)^{0.603} \left( \frac{\rho_s}{\rho_w} \right)^{0.5} \left( \frac{v_s}{s} \right)^{0.5}$</td>
<td>This study</td>
<td>152</td>
<td>144</td>
<td>5</td>
</tr>
<tr>
<td>Solid-block landslide (2018 Anak Krakatau)</td>
<td>$a_{0a}/h = 0.667 \left( \frac{V}{h^2} \right)^{0.8} \left( \frac{\rho_s}{\rho_w} \right)^{0.8} \left( \frac{v_s}{s} \right)^{0.8}$</td>
<td>Bolin et al. (2014)$^b$</td>
<td>134</td>
<td>134</td>
<td>0</td>
</tr>
<tr>
<td>Solid-block landslide (2018 Anak Krakatau)</td>
<td>$a_{0a}/h = 0.25 \left( \frac{V}{h^2} \right)^{0.8} \left( \frac{\rho_s}{\rho_w} \right)^{0.8} \left( \frac{v_s}{s} \right)^{0.8}$</td>
<td>Robbe-Saule et al. (2021)</td>
<td>134</td>
<td>31</td>
<td>77</td>
</tr>
<tr>
<td>Solid-block landslide (2018 Anak Krakatau)</td>
<td>$a_{0a}/h = 0.4545 \left( \frac{\tan \theta}{\tan \phi} \right)^{0.062} \left( \frac{V}{h^2} \right)^{0.296} \left( \frac{\rho_s}{\rho_w} \right)^{0.235}$</td>
<td>This study</td>
<td>134</td>
<td>126</td>
<td>6</td>
</tr>
</tbody>
</table>

Landslide parameters of the 1958 Lituya Bay tsunami are based on Fritz et al. (2004); $v_s = 110$ m/s; $\rho_s = 2700$ kg/m$^3$; $\rho_w = 1000$ kg/m$^3$; and $b_s = 338$ m. For other parameters see Table 3.

Landslide parameters of the 2018 Anak Krakatau event are based on Heidarzadeh et al. (2020), Grilli et al. (2019), and Grilli et al. (2021); $v_s = 44.9$ m/s; $\rho_s = 2300$ kg/m$^3$; $\rho_w = 1000$ kg/m$^3$; $l_s = 1250$ m; $h = 50$ m; and $b_s = 2700$ m. For other parameters see Table 3.

Parameters are $a_{0a}$, initial maximum wave amplitude; $h$, water depth; $\rho_s$, slide density; $\rho_w$, water density; $v_s$, slide velocity; $V$, slide volume; $b_s$, slide width; $l_s$, slide length; $s$, slide thickness; $\alpha$, slope angle; $m_l$, slide mass; and, $V_{wm}$, the volume of the final immersed landslide. We considered $V_{wm} = V$ for both types of landslides.

$^a$ Parameters are $a_{0a}$, initial maximum wave amplitude; $h$, water depth; $\rho_s$, slide density; $\rho_w$, water density; $v_s$, slide velocity; $V$, slide volume; $b_s$, slide width; $l_s$, slide length; $s$, slide thickness; $\alpha$, slope angle; $m_l$, slide mass; and, $V_{wm}$, the volume of the final immersed landslide. We considered $V_{wm} = V$ for both types of landslides.

$^b$ Using Equation (6).

which successfully reproduced the initial wave amplitudes of the 2018 Anak Krakatau (subaerial solid-block), the 1792 Unzen (sub-aerial granular), and the 1958 Lituya Bay (subaerial granular) events. By comparing the performance of our equations with those of the previously-published ones, it is confirmed that our equations perform almost equally or better relative to other existing equations.

CRediT authorship contribution statement

Ramtin Sabeti: Investigation, Methodology, Experiment, Software, Visualization. Mohammad Heidarzadeh: Conceptualization, Methodology, Experiment, Writing – review & editing.
Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The data for this study are described in the article.

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References

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